XVI JORNADAS DE TRABAJO EN MECÁNICA CELESTE
Homenaje a Antonio Elipe en su 60 cumpleaños.

Soria, 19-21 junio 2017
http://albergueweb.uva.es/xvijmc/

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 créditos: NASA.
Las Jornadas de Trabajo en Mecánica Celeste son una iniciativa nacida en 1998 en el seno del Grupo de Mecánica Espacial de la Universidad de Zaragoza que se han convertido en un clásico del panorama científico de España. Hoy en día, estas reuniones permiten a los distintos grupos que trabajan en Mecánica Celeste, bien dentro de la Astronomía, Sistemas Dinámicos o cualquier otra rama del conocimiento, disfrutar de un punto de encuentro periódico, estimulante y bien asentado.

El principal objetivo de este evento es que, durante dos o tres días, los distintos grupos se pongan en contacto, expongán sus líneas actuales de investigación, comenten e intercambien ideas de un modo constructivo. De esta manera, los investigadores noveles tienen a su disposición un foro de expertos donde comunicar sus investigaciones. De la puesta en común han surgido regularmente trabajos y proyectos que más tarde se han integrado en los distintos planes de investigación.

Cada edición de las Jornadas ha sido organizada por una universidad o grupo de investigación diferente. La presente decimosexta edición será acogida por la Universidad de Valladolid, en su Campus de Soria.

Sirvan estas líneas para daros la bienvenida a este entorno que respira la fecundidad del poeta. Que la sombra de Machado nos ayude a elevarnos en nuestras investigaciones.

El comité organizador.
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Lunes 19

09:00 - 09:30 Registro de participantes.
09:30 - 09:45 Presentación de las XVI Jornadas.

Sesión 1: PROBLEMAS DE N-CUERPOS (moderador J.A Docobo).

09:45 - 10:30 Conferencia plenaria: T. Kalvouridis. *N-body tales and other related stories.*

10:30 - 11:00 A. Abad, M. Arribas, M. Palacios, A. Elipe. *Symmetric periodic orbits in the collinear restricted four-body problem.*

11:00 - 11:30 § Pausa café. §

11.30 - 12.00 M. Ollé, O. Rodríguez, J. Soler. *Ejection-collision orbits in the RTBP.*

12:00 - 12:30 M. Corbera. *Orthodiagonal quadrilateral central configurations.*

12:30 - 13:00 M. Álvarez-Ramírez, E. Barrabés. *Ejection-collision orbits in the symmetric collinear four-body problem.*

13:00 - 13:30 J. M. Cors. *On stacked relative equilibria of the 1+4 - body problems.*

13:30 - 15:30 § Comida. §

Sesión 2: SISTEMAS DINÁMICOS (moderador J. Palacián).

15:30 - 16:00 E. Barrabés, J. M. Cors, L. García, M. Ollé. *A dynamical mechanism to explain galactic bridges and tails.*


16:30 - 17:00 A. Daza. *Recent advances in fractal basins boundaries.*

17:00 - 17:30 C. Jaffé, J. Palacián, P. Yanguas, T. Uzer. *Semiclassical quantization of atomic systems: the hydrogen and helium atoms.*

19:30 § Conferencia pública, centro cultural Palacio de la Audiencia §

Antonio Elipe. *Misiones espaciales, qué son y para qué sirven.*
Martes 20

Sesión 3: APLICACIONES (moderador A. Abad).

09:00 - 09:45 Conferencia plenaria: A. Rossi. An holistic approach to the space debris mitigation.

09:45 - 10:10 D. Casanova, M. Dumont, A. Lemaitre, A. Petit. Designing a synthetic population of space debris in the geostationary region.

10:10 - 10:35 C. Abad, E. Lacruz. Astronomía para el proyecto PASAVEN.


11:00 - 11:30 § Pausa café. §


12:20 - 12:45 C. Jaffé. The role of homograohyc dynamics in atomic and molecular systems.


13:30 - 15:30 § Comida. §

15:30 - 16:30 J. A. Docobo, M. Palacios. Sesión especial.

16:30 § Foto de las XVI Jornadas. §

17:30 - 19:30 § Visita guiada a la ciudad. §

Inicio de la visita en el Parador.

21:30 § Cena de las XVI Jornadas. §

La cena tendrá lugar en uno de los salones del Parador.
Programa

XVI JORNADAS DE
TRABAJO EN
MECÁNICA CELESTE

Miércoles 21


09:00 - 09:30  A. Elipe, J. I. Montijano, L. Rández, M. Calvo. On the solution of the hyperbolic Kepler equation by Newton's iterations.

09:30 - 10:00  B. Bardin. On transcendental case in the stability problem of periodic Hamiltonian systems with one degree of freedom.

10:00 - 10:30  S. Breiter, K. Langner. The Kustaanheimo-Stiefel-Lissajous transformation.

10:30 - 11:00  I. Aparicio, L. Floría. On polynomial perturbations of Keplerian systems in focal-type canonical variables.

11:00 - 11.30  § Pausa café. §

11:30 - 12:00  L. Floría. On canonical Frenet Variables.

12:00 - 12:30  F. Martínez de Azagra. Aplicación de la geometría analítica a la geodesía matemática.

12:30 - 13:00  A. Vigueras, J. A. Vera. On the Hamiltonian dynamics of gyrostats and some applications.


13:30  § Clausura de las XVI Jornadas. §

13:45  § Comida. §
1. Resúmenes
Symmetric periodic orbits in a restricted four-body problem

A. Abad\textsuperscript{(1)}, M. Arribas\textsuperscript{(1)}, M. Palacios\textsuperscript{(1)}, A. Elipe\textsuperscript{(1)(2)}

Abstract

We consider a restricted four-body problem where the planar motion of an infinitesimal particle is governed by three primaries rotating in a fixed circular orbit defining a collinear central configuration. The masses of peripheral bodies are equal and a parameter $\beta$ represents the ratio between the mass of the central and the peripheral bodies.

In this paper, we perform a study of the evolution of the families of symmetric periodic orbits when $\beta$ increases from the value $\beta = 0$, which corresponds to the well known Three Body Problem. We use the plane $(x; C)$ ($C$ represents the Jacobi constant) to show how the characteristic curves evolve. When the central body is introduced, some new families appear and others begin to change.

We analyze the evolution of the asymptotic orbits, that connects both triangular points, with respect to the parameter $\beta$. Some families of periodic orbits end at one of these asymptotic orbits. The characteristics curves of the families reach the point in the plane $(x; C)$ corresponding to the asymptotic orbit in a spiral way. We find the value $\beta_{\text{max}}$ for which these spiral points appear in the collinear four body problem; below $\beta_{\text{max}}$, the number of these points goes from two to five, depending on $\beta$, and above it, there are not asymptotic orbits.

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Abstract

El presente trabajo da las bases astrométricas que permiten determinar posiciones astronómicas de los objetos celestes que se observan en una exposición realizada con instrumentos ópticos. Estas posiciones serán utilizadas para la determinación de las efemérides de los satélites artificiales si, como en nuestro caso, son este el tipo de objetos mostrados en la exposiciones, pero que perfectamente podrían extenderse al área de los escombros espaciales. PASAVEN es el acrónimo de Posicionamiento Astrométrico del SAtélite VENESAT1. El marco en el cual me apoyo para la presente exposición da una idea del trabajo, tanto astrométrico como físico que está involucrado en este proyecto, que por estar todavía inacabado no vamos más allá que hacer uso de él para la unión de la charla y en el cual hay un gran grupo de personas involucradas de dos instituciones venezolanas, el Centro de Investigaciones de Astronomía (CIDA) y la Agencia Bolivariana para Asuntos Espaciales (ABAE) a quienes queremos agradecer su participación en el proyecto y que aquí tan sólo se les menciona.

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On Polynomial Perturbations of Keplerian Systems in Focal–Type Canonical Variables

Ignacio Aparicio\(^{(1)}\), Luis Florín\(^{(2)}\)

Abstract

Inspired by Breiter et al. (1996), we considered (Aparicio and Florín, 2000) the application of certain canonical sets of focal–type variables to a class of perturbed Keplerian Hamiltonian systems for which the perturbing potential can be expressed as a polynomial function of the Cartesian coordinates.

As a further development of our previous work, we present a general formulation (and the subsequent reduction to second–order quasi–linear ordinary differential equations with non–constant coefficients, governing a set of four coupled, non–linearly forced oscillators with variable “frequencies”) of this kind of perturbed Kepler problem in terms of generic, unspecified canonical variables of the focal type, introduced by means of a bi–parametric family of canonical transformations extending the projective–factoring point transformation of the configuration space (Aparicio 2013, Ch. 6, §6.2). In this respect, the most significant cases of canonical sets of focal variables, as originally proposed by Ferrándiz and by Deprit, Elipe and Ferrer, can be recovered within the framework of this general approach, as a result of particular instances of the said family of canonical transformations, after appropriate choices of the numerical values of the parameters (Aparicio 2013, Ch. 6, §6.6).

References


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On Transcendental Case in the Stability Problem of Periodic Hamiltonian Systems with One Degree of Freedom

Boris Bardin$^{(1)}$

Abstract

The most rigorous and full results in the stability problem of periodic Hamiltonian systems with one degree of freedom have been recently obtained in [1, 2]. By using these results one can reduce the stability problem of original Hamiltonian system to a stability study of an autonomous truncated system, whose Hamiltonian is a part of the series expansion for the original Hamiltonian function normalized up to terms of a finite order. However, there are examples when the stability problem cannot be solve by taking into account terms of any finite order. In this situation the results of papers mentioned cannot be applied and we say that the so-called transcendental case in the stability problem takes place. We consider the transcendental case which appears in presence of the first order resonance. We establish necessary and sufficient conditions for existence of the transcendental case as well as show that the Hamiltonian system is unstable in this case. We also illustrate our results in some problems of classical mechanics. The work was carried out under the project of the Ministry of Education and Science of the Russian Federation (No 3.3858.217/ProjectPart).

References


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Ejection-Collision orbits in the symmetric collinear four body problem

M. Álvarez-Ramírez\(^{(1)}\), E. Barrabés\(^{(2)}\) Mercè Ollé\(^{(3)}\)

Abstract

We consider a collinear symmetric four body problem, in which four masses \(m_2\), \(m_1\), \(m_1\) and \(m_2\) are in collinear configuration (in this order) and move symmetrically about the center of mass. The problem has been studied by different authors in the last decades. In [4], a wide numerical exploration is done, showing different type of orbits: a Shubart-like periodic orbit, quasiperiodic orbits and scattering orbits. In [3], the authors explore, for the case of equal masses, the global geometry of the phase space via a suitable Poincaré map. In [1], the authors study analytically the quadruple collision, as well as the infinity, by means of the McGehee’s techniques.

We will revise the mentioned works and will show how the study of the invariant manifolds associated to the equilibrium points inside the collision manifold allow us to complete them. In particular, we will focus on the ejection-collision orbits, that is, orbits that tend forwards and backwards in time to the quadruple collision, which can be classified depending on the number and type of intermediate binary collisions. In order to do so, we parameterize the invariant manifolds up to a desired order using the parametrization method ([2]).

References


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The Kustaanheimo-Stiefel-Lissajous transformation

Sławomir Breiter\(^{(1)}\), Krzysztof Langner\(^{(1)}\)

Abstract

We extend the definition of the Kustaanheimo-Stiefel (KS) variables, introducing the notion of a defining vector. The defining vector makes explicit the presence of some distinguished 3D space axis in the KS transformation, which traditionally has been either the first, or the third axis of the Cartesian reference system. Actually, the defining vector should be chosen in accord with existing symmetries. The link between the first integrals of the resulting 4D harmonic oscillator (including the Fradkin tensor) and the Keplerian first integrals is explicitly established. Then, thanks to two sets of the Lissajous variables, the transformed Hamiltonian is split into a three degrees of freedom oscillator and an ignorable conjugate pair of action-angle variables. The latter refers to the fibration property of the KS transformation. The work applies or extends the ideas and techniques developed by Deprit, Elipe and Ferrer for the KS transformation, and by Deprit, Elipe, Miller and Williams for the Lissajous variables.
Binaries as a source of information of stellar masses and precise parallaxes in the GAIA era

José Ángel Docobo\textsuperscript{(1)}, Pedro Pablo Campo\textsuperscript{(1)}, Vakhtang Tamazian\textsuperscript{(1)}

Abstract

It is possible to calculate simultaneously both the visual and spectroscopic orbits of binary systems by means of high-resolution techniques and spectroscopy, and from them we can to obtain not only the masses and parallaxes with high accuracy, but also the three-dimensional orbit.

The parallaxes obtained with this method (orbital parallaxes) are fundamental to check the values measured by astrometric missions such as GAIA.

In this communication we comment the last cases studied in the Observatorio Astronómico Ramón María Aller of the Universidade de Santiago de Compostela in collaboration with the Observatory of the Institute of Astronomy from the University of Cambridge.

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Designing a synthetic population of space debris in the geostationary region

Daniel Casanova\(^{(1)}\), Morgane Dumont\(^{(2)}\), Anne Lemaitre\(^{(2)}\) and Alexis Petit\(^{(2)}\)

Abstract

Since the beginning of the space era, the number of debris has increased drastically. Governments and space agencies are taking action to reduce catastrophic events, and are making efforts to better know the orbital environment by means of radar and optical observations. In the Geostationary (GEO) region we can observe and track objects with a size about 1 meter by telescope means. However, a huge population of space debris still remain unknown for us.

In this work, we propose to generate a synthetic (artificial) population of individual space debris, whose global characteristics are the same as the real one. To design this population of space debris, we proceed as follows: first, we create a cloud of space debris using a combination of propagators, fragmentation models and historical data creating a simulated population of space debris, similar to the real one. After that, we apply an Iterative Proportional Fitting (IPF) method to the simulated population to include additional pieces of space debris. Consequently, the new population will have two different types of pieces of debris; one type of pieces are generated through the simulation, the other type of pieces are created thanks to the IPF method. Finally, we will have a synthetic population of space debris whose characteristics will be similar to the real population.

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Orthodiagonal quadrilateral central configurations

Montserrat Corbera\textsuperscript{(1)}

Abstract

An orthodiagonal quadrilateral is a quadrilateral in which the diagonals cross perpendicularly. In this talk we prove that any orthodiagonal quadrilateral central configuration of the four-body problem is necessarily a kite. The result is proved for general power-law potential functions as well as for the planar four-vortex problem. The techniques used here can be applied to other similar problems.

This is a joint work with Josep M. Cors (UPC) and Gareth Roberts (Holly Cross College).

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On stacked relative equilibria of the $1+4$–body problem

Josep M. Cors$^{(1)}$

Abstract

We study the relative equilibria of the limit case of the planar Newtonian 5–body problem when four masses tend to zero. In general, it is possible that several of the small masses coalesce in the limit. We call the other equilibria the relative equilibria of the $1+4$–body problem. We focus our interest in the case where two satellites and the large body limit to an equilateral triangle configuration. The equilateral triangle is a relative equilibria in the three body problem, so we call these configurations stacked relative equilibria.

Since the limits of relative equilibria as the small masses or satellites tend to zero have all four small masses on the unit circle and the problem is rotationally symmetric, we assume that the large mass is at the origin, one small mass limits to the point $(1,0)$ and one small mass limits to $(\cos(\pi/3), \sin(\pi/3))$. It is easy to show that relative equilibria do not occur when the other two satellites limit to points in the arc from zero to $\pi/3$. When at least one small mass limit to a point outside the arc from zero to $\pi/3$ then there exist a relation among the angles such that the resulting configuration is a relative equilibria for a two parameter family of masses.

This is a work in progress, joint with Diana Atanasova, Roderic Guigo and Dick Hall, from Boston University.

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Recent advances in fractal basins boundaries

Álvar Daza\textsuperscript{(1)}

Abstract

Basins of attraction take its name from hydrology, and in dynamical systems they refer to the set of initial conditions that lead to a particular final state. When different final states are possible, the predictability of the system depends on the structure of these basins. A very special case of fractal basins arises when three or more basins share a common boundary, a counter-intuitive situation known as Wada basins. We will revise recent works that propose new tests for the Wada property and we will try to answer an apparently simple question: Are Wada basins more unpredictable than other fractal basins? This will lead us to the concept of basin entropy.

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Abstract

The aim of a repeating groundtrack orbit is to periodically revisit areas of observation. To achieve it, a commensurability between the satellite nodal period and the period of the Earth’s rotation is established. Nevertheless, the classical procedure only includes the oblateness term $J_2$. This work examines the design of repeat groundtrack orbits and presents a way to improve the maintenance of the repeating property subjected to other orbital perturbations. The model takes into account a series of orbital perturbations such as the gravitational potential of the Earth, the atmospheric drag, the Sun and the Moon as disturbing third bodies and the solar radiation pressure. This correction scheme can be implemented to design satellite constellations in such a way that all the satellites benefit of repeating groundtrack property. Finally, we analyze the addition of other orbital properties such as Sun-synchrony in the design of the constellation.
On Keplerian reductions through orbital elements

S. Ferrer (1), F. Crespo (2)

Abstract

We address the task of using orbital elements to read into invariants and reductions of Keplerian systems from an astrodynamical point of view. New local coordinates for the orbit space $S^2_L \times S^2_L$ of bounded Keplerian orbits are presented. Their principal feature is that they ‘separate’ the orbital plane from the Laplace vector in that plane. They are an alternative to the classical coordinates defined by the linear combination of the angular momentum and Laplace vectors. In the presence of an axial symmetry, these coordinates illustrate in a straightforward manner the singular reduction. For the twice reduced space $\Gamma_{L,H}$, based on the our choice of coordinates for the first reduced space, we propose a variant to the set of coordinates given by Cushman. We finish with two examples.

1.13. Ferrer, S. (Indice)
On Canonical Frenet Variables

Luis Floría

Abstract

Javier Ribera and Antonio Elipe ("Keplerian problems in Frenet Variables". Revista de la Academia de Ciencias Exactas, Físicas, Químicas y Naturales de Zaragoza, Serie 2ª, Vol. 56, (2001), pp. 63–68) proposed a set of canonical variables based on the intrinsic coordinates (analogous to the Cartesian ones) pertaining to the three orthogonal directions of the moving Frenet–Serret trihedron of the Classical Differential Geometry of Curves in ordinary, three-dimensional space. Those coordinates were then completed with their canonically conjugate momenta so as to obtain a set of canonical variables that these authors called Frenet variables.

As a first step, they considered the apsidal frame corresponding to the Keplerian motion of a particle of unit mass about an attracting centre of force, and Cartesian coordinates (with respect to this apsidal frame with its origin at the centre of attraction) within the plane of motion. They also considered the moving Frenet–Serret trihedron at an arbitrary point of the Keplerian orbit at issue, and the Cartesian coordinates with respect to the axes defined by the binormal and tangent directions at that position of the particle. These spatial reference frames are related to each other by a rotation about the (common) axis perpendicular to the plane of motion, and the coordinate transformation can be made a time-independent, completely canonical transformation which is, in particular, a Mathieu transformation.

In this contributed paper we provide some additional details on the definition and theoretical properties of this kind of canonical set studied by Ribera and Elipe.
The role of homographic dynamics in atomic and molecular systems

Charles Jaffé

Abstract

Homographic dynamics are known to play an important role in many Newtonian systems. They are regularly used in the design of the orbit of space craft. Much less is known about the occurrence, existence and role that the homographic dynamics play in Coulombic systems. In this talk I will discuss the role of the homographic dynamics in helium atom He and in the hydrogen cation H$_2^+$. 
Abstract

The regular polygon problem of $N + 1$ bodies (also known as the ring problem of $N + 1$ bodies), is a theoretical model in the area of Celestial Mechanics, aiming to approach the dynamical behavior of a small body, either natural or artificial, under the effect of $N$ big bodies (primaries) the $\nu = N - 1$ of which have equal masses and are located at the vertices of a regular polygon, while the $N$th body with a different mass is located at the center of mass of the system. The particular model has mainly been studied during the last 18 years both in its original classical form and in various versions, by an increasing number of researchers. The configuration of the $N$ big bodies was based on a model proposed by Maxwell in the middle of the 19th century who tried to explain the already known rings of Saturn. Later the model was used to simulate planetary systems with co-orbital satellites (moons). Here we note that quasi co-orbital systems have been observed in the outer planets of our solar system. My speech is a combination of the principal historical milestones of the problem, a brief reference to some results obtained so far, an overview of the recent research and some new versions of the problem aiming to improve the original formulation.
Stability of the permanent rotations of an asymmetric gyrostat in a uniform Newtonian field

V. Lanchares$^{(1)}$, A.I. Pascual$^{(1)}$, M. Iñarrea$^{(1)}$, A. Elipe$^{(1)}$

Abstract

The stability of the permanent rotations of a heavy gyrostat is analyzed by means of the Energy-Casimir method. Sufficient and necessary conditions are established for some of the permanent rotations. The geometry of the gyrostat and the value of the gyrostatic moment are relevant in order to get stable permanent rotations. Moreover, the necessary conditions are also sufficient, for some configurations of the gyrostat.

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Aplicación de la Geometría Analítica a la Geodesia Matemática

Fernando Martínez de Azagra

Abstract

Las técnicas de la Geodesia Matemática han tenido que hacer frente a dos grandes condicionantes: por una parte la complejidad del elipsoide y por otra la necesidad de elaborar unas fórmulas, que pudieran calcularse con los medios disponibles en cada momento, de manera que la potencia informática actual nos permite ser mucho más modestos en la estructuración de las fórmulas, a cambio de perder la belleza intelectual de las ideas de antaño. Por otro lado, la tecnología del GPS ha relegado a la Geodesia Matemática clásica a un segundo plano, aunque esta realidad no significa su obsolescencia. Entre otras razones porque existen muchos elementos comunes entre la Geodesia terrestre y la espacial, de manera que la primera puede servir de base para el estudio del posicionamiento desde los satélites artificiales, siempre que seamos capaces de organizar un mismo planteamiento matemático común a ambas ramas.

Sin embargo, en los tiempos actuales las Geodesias Matemática y Espacial siguen unos caminos totalmente diferentes, ya que la primera conserva su forma original, mientras que la segunda utiliza la geometría cartesiana de un modo dominante. Pues bien, la Geodesia basada en las mediciones terrestres de ángulos o de distancias, también puede resolverse utilizando un sistema cartesiano de referencia anclado a nuestro planeta. Como la Geodesia terrestre cartesiana prácticamente no se utiliza, su estudio puede convertirse en una línea de investigación, en un terreno casi inexplorado. Esta circunstancia nos ha permitido diseñar el “Método de los Planos” con un alto grado de libertad y autonomía. Esperemos pues que, en un futuro próximo, esta forma de entender la Geodesia ocupe un puesto importante, tanto en sus aplicaciones técnicas como en su utilización teórica en una fase previa al estudio del posicionamiento mediante satélites.
Differential Galois Theory and non-integrability of planar polynomial vector fields

Juan J. Morales-Ruiz

Abstract

We study a necessary condition for the integrability of the polynomials fields in the plane by means of the differential Galois theory. More concretely, as a corollary of a previous result with Ramis and Simó on Hamiltonian systems, it is proved that a necessary condition for the existence of a meromorphic first integral is that the identity component of the Galois group of the higher order variational equations around a particular solution must be abelian. We illustrate this theorem with several families of examples. A key point in these applications is to check whether a suitable primitive is elementary or not. Using a theorem by Liouville, the problem is equivalent to the existence of a rational solution of a certain first order linear equation, the Risch equation. This is a classical problem studied by Risch in 1969, and the solution is given by the “Risch algorithm”. In this way we point out the connection of the non integrability with some higher transcendent functions, like the error function.

(This is a joint work with Primitivo B. Acosta-Humánez, J. Tomás Lázaro and Chara Pantazi)

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The Theory of Connections with Applications

Daniele Mortari

Abstract

This study introduces a procedure to obtain general expressions, $y = f(t)$, subject to linear constraints on the function and its derivatives defined at specified values [1]. These constrained expressions can be used to describe all possible trajectories satisfying specific constraints. This capability allows to obtain general expressions to solve linear differential equations [2] with no more need to satisfy additional constraints (the “subject to:” expressions) as all the constraints are embedded in the constrained expression. In particular, for expressions passing through a set of points, a generalization of the Waring’s interpolation form, is introduced. The study shows how to use these constrained expressions to obtain least-squares solutions to initial and boundary value problems to nonhomogeneous linear differential equations with nonconstant coefficients of any order. The proposed method has two steps. The first step consists of writing the constrained expressions, previously introduced, that has embedded the differential equation constraints. These expressions are given in terms of a new generic function, $g(t)$, and they satisfy the constraints, no matter what $g(t)$ is. The second step consists of expressing $g(t)$ as a linear combination of a set of independent basis functions. Specifically, four different basis functions are considered: 1) Monomials, Fourier series, Chebyshev orthogonal polynomials of the first kind, and Legendre polynomials. The procedure leads to a set of linear equations in terms of the unknown coefficients vector that is then computed by least-squares. Numerical examples are given to quantify the solutions accuracies and the results comparisons using different basis functions.

References


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A dynamical mechanism to explain galactic bridges and tails

Esther Barrabés(1), Josep Maria Cors(1), Laura Garcia(1) and Mercè Ollé(2)

Abstract

After a close encounter of two galaxies, bridges and tails can be seen between or around them. A bridge would be an spiral arm between a galaxy and its companion, whereas a tail would correspond to a long and curving set of stars escaping from the galaxy. The goal of this paper is to present a mechanism, applying techniques of dynamical systems theory, that explains the formation of tails and bridges between galaxies in a simple model, that is, assuming that such galaxies are two primaries describing parabolic orbits, the so called Restricted parabolic three-body problem. We show that the invariant manifolds of the collinear equilibrium points and the ones of the collision manifold explain the formation of bridges and tails.

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On the solution of the hyperbolic Kepler equation by Newton’s iterations

A. Elipe(1), J.I. Montijano(2), L. Rández(2), M. Calvo(2)

Abstract

In this paper the determination of low computational cost starters \( S_0 = S_0(L, g) \) for the iterative solution of the hyperbolic sine Kepler equation by means of Newton’s method is considered. This equation is written in the form

\[
F(S; L, g) \equiv S - g \text{Arcsinh}(S) - L = 0, \quad (L, g) \in [0, +\infty) \times (0, 1),
\]

where \( g = 1/e \in (0, 1) \) is the inverse eccentricity of the hyperbolic orbit, \( L = gM \) , (\( M \) the mean anomaly) and the unknown is \( S = \sinh(H) \).

First of all, several improvements over the piecewise-defined starter function \( S_0 = S_0(L, g) \), \( (L, g) \in [0, +\infty) \times (0, 1) \) proposed by Avendaño, Martín-Molina and Ortigas-Galindo in (Cel. Mech. Dyn. Astron. (2015), 123:435–451) are considered to reduce some measures of the defect in the first Newton’s iteration \( |N_F(S_0(L, g)) - S_0(L, g)| \) where \( N_F(x) \equiv x - F(x; L, g)/F'(x; L, g) \) is the Newton’s iteration function of \( F \). In addition, the super–convergence property \( |S_{n+1} - S_{n}| \leq q^{2^n-1}|S_1 - S_0| \) that holds for all \( n = 0, 1, \ldots \) with rate \( q = 1/2 \) implied by Smale’s \( \alpha \)-test is improved in the sense that for the rate \( q \) a function \( q = q(g) \) in terms of the inverse eccentricity \( g \in (0, 1) \) is derived so that \( q(g) \rightarrow 0 \) when \( g \rightarrow 0 \) showing that convergence is much faster when \( g \) is close to zero.

Secondly, taking into account the monotonic increasing and convexity of \( F(S; L, g) \) as a function of \( S \), a piecewise-defined monotonic starter function \( S_0 = S_0(L; g) \) that satisfies the \( \alpha \)-test with low computational cost is constructed. In this case the sequence generated from each \( S_0 = S_0(L; g) \) decreases monotonically to the unique exact solution of \( F(S; L, g) = 0 \) for all \( L > 0 \) and \( g \in (0, 1) \).

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Ejection-collision orbits in the RTBP

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Abstract

As it is well known, for any value of the mass parameter \( \mu \in (0,0.5] \) and sufficiently restricted Hill regions, there are exactly four ejection-collision orbits. We check their existence and extend numerically these four orbits for \( \mu \in (0,0.5] \) and for less restrictive values of the Jacobi constant.

We consider \( n \)-ejection-collision orbits, we explore them numerically for \( \mu \in (0,0.5] \) and suitable values of the Jacobi constant and we discuss the appearing bifurcations.

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An holistic approach to the space debris mitigation

Alessandro Rossi(1)

Abstract

As mentioned in the H2020 Protec 2015 call, the impact of orbital debris on the space activities should be reduced tackling the problem from different points of view, including prevention, mitigation and protection.

The ReDSHIFT (Revolutionary Design of Spacecraft through Holistic Integration of Future Technologies) project, approved by the European Community, is trying to achieve these goals through a holistic approach that considers, from the outset, opposing and challenging constraints for the space environment preservation, the spacecraft survivability in the harsh space environment and the safety of humans on ground.

The main innovative aspects of the project concern a synergy between theoretical and experimental aspects, such as: long term simulations, astrodynamics, passive de-orbiting devices, 3D printing, design for demise, hypervelocity impact testing, legal and normative issues. The talk presents a quick overview of the first ReDSHIFT results in an effort to highlight the holistic approach of the project covering different aspects of the space debris mitigation field. Particular emphasis will be devoted to the theoretical results obtained in the long term simulation studies and on the mapping of the phase space around the Earth, from the LEO (Low Earth Orbit) to the GEO (Gestationary Orbit) region, studying the stability and instability areas. The idea here is to exploit the former as, possibly interim solutions, graveyard zones and the latter as “highways” to de-orbit, easing and, hence, enhancing the compliance to the proposed de-orbiting mitigation measures. The mapping shall be exploited by proposing affordable maneuvers to move the spacecraft at the end-of-life towards the most favorable regions of the phase space. This includes also the study of the possibility to use passive means to de-orbit, such as area augmentation devices (solar and drag sails).

ReDSHIFT is now entering its second year of work. In the coming months the main efforts will be devoted to the design, 3D printing and testing aspects.

The research leading to these results has received funding from the Horizon 2020 Program of the European Union’s Framework Programme for Research and Innovation (H2020-PROTEC-2015) under REA grant agreement n° [687500]- ReDSHIFT.

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A nonlinear-dynamical approach to the formation of the RbCs dimer by a laser pulse

J. Pablo Salas\(^{(1)}\), C. Chandre\(^{(2)}\), Jorge Mahecha\(^{(3)}\)

Abstract

We present in the talk the results of the study of the formation of the RbCs molecule by an intense laser pulse using nonlinear dynamics. The system is modeled by a two-degree-of-freedom rovibrational Hamiltonian, which includes the ground electronic potential energy curve of the diatomic molecule and the interaction of the molecular polarizability with the electric field of the laser. As the laser intensity increases, we observe that the formation probability first increases and then decreases after reaching a maximum. We show that the analysis can be simplified to the investigation of the long-range interaction between the two atoms. We conclude that the formation is due to a very small change in the radial momentum of the dimer induced by the laser pulse. From this observation, we build a reduced one-dimensional model which allows us to derive an approximate expression of the formation probability as a function of the laser intensity.

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On the Hamiltonian dynamics of gyrostats and some of its applications

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Abstract

In this paper, we briefly review some of the various problems addressed in our working group about the movement of gyrostats and applications. We address the problems basically through the canonical or non-canonical Hamiltonian formulation.

First, we consider the motion of a gyrostat with a fixed point in a central Newtonian force field, we integrate this problem in some particular cases through the Hamilton-Jacobi equation or by means of the first integrals of the problem, also we propose to obtain the action-angle variables and study the relative equilibria and their Lyapunov stability.

Then, we study the rotational motion of a gyrostatic satellite in an elliptical orbit with or without inclination, using the Lie-Deprit canonical perturbations method, and apply it to the study of the Earth’s rotation, considering the Earth as a gyrostat.

Finally, we consider the general case of motion of a gyrostat attracted by n spherical rigid bodies, in particular for n = 1, 2, etc., using the non-canonical Hamiltonian formulation, by means of geometric-mechanics methods, relative equilibria are obtained in such problems and its stability was studied by the Energy-Casimir method.

In addition, we also point out some open problems.

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Semiclassical quantization of atomic systems through their normal form: The hydrogen and helium atoms

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Abstract

Over a century after Bohr’s the initial quantization of hydrogen, the semiclassical quantization of atomic systems still represents a challenge. In the present communication we re-examine the semiclassical quantization of hydrogen asking the question: How can hydrogen be quantized without making use of its separability? The approach adopted was to explicitly construct a transformation from the physical variables to the action-angle variables. The initial difficulty encountered is the lack of an equilibrium point on the potential energy surface. To surmount this difficulty, it is noted that the circular periodic orbits are as relative equilibria. In a rotating frame the relative equilibria become critical points in the phase flow. It is shown that the flow in the vicinity of the critical point is stable. The Lie-Deprit transformation is then used to transform the system into normal form, following which the semiclassical quantization is straightforward. The helium is considered as a classical spatial Coulomb three body problem. We follow the same approach with the difference that the computations in this case are more involved and the equilibrium is of saddle type. We apply transition state theory by computing the dividing surface around the saddle and the corresponding invariant manifolds.

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